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13. ABSTRACT *(Maximum 200 words)* This project involved the study of the control and dynamics of various physical and engineering systems.

The Principal Investigator analyzed the stability of mechanical systems in the presence of dissipation, as well as the stabilization of mechanical systems by using nonlinear controls. He studied in particular a method of control that involves matching a feedback controlled system by an autonomous controlled Lagrangian system by adjusting parameters. He analyzed control of satellite dynamics by this method. He studied the geometry, control and stabilization of systems with nonholonomic constraints - systems such as wheeled vehicles or contour following robots. He derived an energy-based method for analyzing such nonholonomic systems, even in the case when the system has natural dissipation. He also studied the role of conservation laws in nonholonomic systems. He studied optimal control problems and, in particular, solvable optimal control problems, and derived a novel form of the equations for the rigid body using the optimal control approach. These equations were linked with discrete rigid body equations and numerical analysis. He also worked on the stabilization of systems with complex dynamics arising in anti-corrosion processes.

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Summary of Completed Project

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1 Technical Information.

In work with J. Marsden and G. Sanchez (Bloch, Marsden and Sanchez [1997]) and more recently with N. Leonard (Bloch, Leonard and Marsden [1997, 1998, 1998a]), I analyed a technique for stabilizing nonlinear systems using the notion of controlled Lagrangian. The procedure is as follows: we are given a mechanical system with symmetry and a relative equilibrium that we wish to stabilize. Examples include a spacecraft with momentum wheels or a pendulum on a cart. We then construct a new Lagrangian that we call $L_{\tau,\rho,\sigma}$ which describes the closed loop behavior of the controlled system, but which still preserves a modified energy and momentum. Various parameters are adjusted, in particular the kinetic energy metric, in order to ensure the controlled Lagrangian matches the real physical system. Since energy and momentum are preserved, we are able to use the energy-momentum method to analyze stability.

For example, the Lagrangian for a free spacecraft rotor system is of the form $L = 1/2(\lambda_1\omega_1^2 + \lambda_2\omega_1^2 + I_3\omega_3^2 + J_3(\omega_3 + \dot{\alpha})^2)$.

We designed a controlled Lagrangian of the form

$$L_{\tau,\sigma,\rho} = 1/2(\lambda_1\omega_1^2 + \lambda_2\omega_1^2 + I_3\omega_3^2 + \rho J_3((1+r)\omega_3 + \dot{\alpha})^2) + J_3\sigma r^2\Omega_3^2) \,.$$

A suitable choice of parameters matches the equations of the controlled Lagrangian with the equations of the physical controlled system. Further, these equations are still Hamiltonian and preserve a momentum like quantity. Thus by adjusting the control parameters we can use energy methods to achieve stabilization results.

One general class of Lagrangian system that we showed we can stabilize with this technique (the spherical pendulum on a cart and the nonlinear pendulum are special cases) is of the following form:

Locally, we write coordinates for our configuration space Q as x^α, θ^a where $x^\alpha, \alpha = 1, \ldots n$ are coordinates on the shape space Q/G for G an Abelian group, and where $\theta^a, a = 1, \ldots, r$ are coordinates for the group. For the uncontrolled system, the variables θ^a will be cyclic coordinates in the classical sense. We write the given Lagrangian in these coordinates (with the summation convention in force) as

$$L(x^\alpha, \dot{x}^\beta, \theta^a, \dot{\theta}^b) = \frac{1}{2}g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta + g_{\alpha a}\dot{x}^\alpha\dot{\theta}^a + \frac{1}{2}g_{ab}\dot{\theta}^a\dot{\theta}^b - V(x^\alpha) \,. \tag{1.1}$$

We now introduce a shift in corresponding velocity variables by a 1-form $\tau = \tau_{a\alpha}\dot{x}^\alpha$.

The controlled Lagrangian is then

$$L_{\tau,\sigma} = L(x^\alpha, \dot{x}^\alpha, \dot{\theta}^a + \tau^a_\alpha\dot{x}^\alpha) + \frac{1}{2}\sigma_{ab}\tau^a_\alpha\tau^b_\beta\dot{x}^\alpha\dot{x}^\beta \tag{1.2}$$

We can then show that a system with suitable controls in the θ variables can be modeled by this *autonomous* Lagrangian. Interestingly, the structure of the controls mirrors the structure of the system inertia matrix.

With my student Lum (Lum and Bloch [1997], 1998]) I analyzed the control and numerics of satellite dynamics in the so-called Serret-Andoyer variables. We showed our methods were advantageous both in simplifying the theoretical analysis and in reducing computational work. With another student Rui and N. McClamroch we derived controls for orienting coupled rigid bodies.

With Marsden, Zenkov and Murray I studied the geometry and dynamics of mechanical systems with nonholonomic constraints. Such constraints arise in many important mechanical systems including robotic systems. The resulting equations are not variational in nature but arise from the Lagrange D'Alembert formalism. In many cases the equations of motion are of the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}^i}\right) - \frac{\partial L_c}{\partial r^i} = \left(\frac{\partial L}{\partial s^a}\right) B^a_{ij}\dot{r}^j$$

where L_c is the Lagrangian with the constraints substituted and the B's are curvature terms. Given a symmetry in the problem and defining the nonholonomic momentum map to be $J^c_\xi = \frac{\partial L}{\partial \dot{q}^i}(\xi^q_Q)^i$, we showed one gets generalized conservation laws of the form

$$\frac{dJ^c_\xi}{dt} = \frac{\partial L}{\partial \dot{q}^i}\left[\frac{d}{dt}(\xi^q)\right]^i_Q.$$

Here ξ^q_Q is the infinitesimal generator of the symmetry group and ξ^q is a configuration dependent Lie algebra element that measures how the distribution defined by the constraints and the group orbit interact. This nonholonomic momentum map gives a rather beautiful generalization of Noether's theorem to the nonholonomic context. It indicates that in some cases one obtains conservation laws as in the Hamiltonian case, but in other cases one obtains a dynamic momentum equation which is in fact key to many aspects of the dynamic behavior of the system.

With J. Marsden and my student D. Zenkov ([1998]), I analyzed the stability properties of such systems. In particular we have formulated an energy momentum approach to the analysis of such systems, i.e. we showed we could find a Lyapunov function of the form $V = E + \Phi$ where E is the energy and Φ is another conserved quantity. In view of the nontrivial nature of the momentum equation, Φ is not the momentum in general but a quantity which arises more subtly from the analysis of the momentum equation. In some cases one can get asymptotic stability, even in the absence of external dissipation. An example that exhibits the latter behavior is the so-called rattleback top, while the rolling disc or penny exhibits stability but not asymptotic stability.

We showed it was useful to divide the energy momentum analysis into three cases which exhibit rather different dynamic behavior:

1. **Pure Transport Case** In this case, terms quadratic in internal (shape) variables are not present in the momentum equation, so it is in the form of a transport equation. Under certain integrability conditions the transport equation defines

invariant surfaces (not momentum level sets), which allowed us to use a type of energy-momentum method for stability analysis in a similar fashion to the holonomic case. In this case one gets stable, but not asymptotically stable, relative equilibria. Examples include the rolling disk, a body of revolution rolling on a horizontal plane and Routh's problem.

2. **Integrable Transport Case** In this case terms quadratic in the shape variables are present in the momentum equation but the transport part is integrable. Then the relative equilibria may be asymptotically stable. We were able to find a generalization of the energy-momentum method which gives conditions for asymptotic stability. An example is the so-called roller racer (two connected carts on the plane).

3. **Nonintegrable Transport Case** Again quadratic terms are present in the momentum equation but the transport part is not integrable. We were able to demonstrate asymptotic stability under certain eigenvalue hypotheses. An example is the rattleback top.

An instructive example which exhibits dissipative behavior is the Lagrangian on $T\mathbb{R}^3$ of the form

$$L(r^1,r^2,s,\dot{r}^1,\dot{r}^2,\dot{s}) = \frac{1}{2}\{(1-[a(r^1)]^2)(\dot{r}^1)^2-2a(r^1)b(r^1)\dot{r}^1\dot{r}^2+(1-[b(r^1)]^2)(\dot{r}^2)^2+\dot{s}^2\}-V(r^1)$$

where a, b, and V are given real valued functions of a single variable and with the nonholonomic constraint $\dot{s} = a(r^1)\dot{r}^1 + b(r^1)\dot{r}^2$. By constructing a suitable Lyapunov function we can show this system has an equilibrium which is asymptotically stable in certain directions even though energy is preserved.

In our recent paper, Bloch and Crouch [1998], we investigated the structure of the connections describing nonholonomic systems on Riemannian manifolds and the relationship of the flows with conserved integrals.

With Drakunov (see e.g. Bloch and Drakunov [1996], [1998]), I studyed stabilization of nonholonomic control systems by nonsmooth feedback (such systems are never smoothly stabilizable). Using sliding mode type techniques we have been able to design a stabilizer for Brockett's fundamental example (the Heisenberg system) with equations $\dot{x} = u$, $\dot{y} = v$, $\dot{z} = xv - yu$.

We designed a series of control laws for this system, for example, $u = -x + ysign(z)$, $v = -y - xsign(z)$. We showed that with this feedback the condition for the system to be stabilized is $1/2[x^2(0) + y^2(0)] \geq |z(0)|$. This is a paraboloid in the phase space, but within the paraboloid we showed that one can use constant feedback to emerge from the paraboloid, thus giving a globally stabilizing controller.

We have extended this analysis to systems in the canonical form of Brockett (see Bloch and Drakunov [1998])

$$\begin{aligned} \dot{x} &= u \\ \dot{Y} &= xu^T - ux^T \end{aligned}$$

where x is a vector and Y is a skew symmetric matrix. For this we use a hybrid control.

Moreover (see Bloch, Drakunov and Kinyon [1997], [1998]) we were able to extend these ideas to a Lie algebraic generalization of this system. The general approach involves reduction of this high order system to a low order one and uses symmetries of the controlled vector fields. Recently we have developed an approach based on switching between isospectral matrix flows and gradient-like double bracket flows. This gives a new method for stabilizing a large class of mechanical systems with constraints.

The general system we studied can be described as follows. Let $\mathfrak{g}$ be a Lie algebra. Assume $\mathfrak{g}$ has a direct sum decomposition $\mathfrak{g} = \mathfrak{m} + \mathfrak{h}$ such that $\mathfrak{h}$ is a Lie subalgebra, $[\mathfrak{h}, \mathfrak{m}] \subseteq \mathfrak{m}$, and $[\mathfrak{m}, \mathfrak{m}] = \mathfrak{h}$. We consider the following system in $\mathfrak{g}$:

$$\begin{aligned} \dot{x} &= u && (1.3) \\ \dot{Y} &= [u, x] && (1.4) \end{aligned}$$

where $x, u \in \mathfrak{m}$, $Y \in \mathfrak{h}$.

This is a canonical form for a large class of systems of interest and has many applications. The key controls we consider are given by

$$u = -\alpha x + \beta[Y, x] - \gamma[Y, [Y, x]] \tag{1.5}$$

where $\alpha, \beta, \gamma : \mathfrak{g} \to \mathbb{R}$ are real-valued ad $\mathfrak{h}$-invariant functions

The idea of the control is to switch between having α nonzero and β zero and vice versa. This alternately drives x or Y downward while the other set of variables evolves isospectrally. In fact one alternates between Lax type equations and a kind of double bracket equation of the type used in optimization problems. Our strategy for control here is a rather nice combination of our work on optimization (double bracket equations), integrable systems (isospectral flows) and our previous work on nonholonomic systems.

With P. Crouch and R. Brockett I have been studying explicit solutions to a number of other optimal control problems of interest – see Bloch and Crouch [1996a] [1997] and Bloch, Brockett and Crouch [1997]. These control problems yield equations of motion which are explicitly solvable – giving for example the equation of motion of the rigid body equations and the Toda lattice. In some cases we have shown that they take the coupled double double bracket form:

$$\begin{aligned} \dot{Q} &= [Q, J^{-1}[P, Q]] \\ \dot{P} &= [P, J^{-1}[P, Q]]\,. && (1.6) \end{aligned}$$

These equations in fact represent the geodesic flow on an adjoint orbit of a compact Lie group with respect to a right invariant metric.

Unlike the double bracket equation $\dot{L} = [L, [L, N]]$ which is gradient and useful for solving discrete optimization problems, the double double bracket equations are Hamiltonian and useful for certain optimal control problems. Further, this approach sheds light on the problem of explicitly solving certain classical problems in dynamics:

The equations discussed above are also well defined on the complex and real Grassmannians of q-planes in $n+1$-space $G_{q,n+1}(\mathbb{C})$ or $G_{q,n+1}(\mathbb{R})$. This may be seen as follows:

We represent a point in the complex (real) Grassmannian by a matrix

$$\hat{Q} = \begin{bmatrix} 0 & Q \\ -Q^* & 0 \end{bmatrix} \tag{1.7}$$

in m where Q is a complex (real) $p \times q$ matrix of full rank and Q^* is its Hermitian conjugate (transpose). Then geodesic equations on the real or complex Grassmannian take exactly the same form as in 1.6 but where Q is replaced by $\hat{Q}$ and similarly for P.

We were able to use formalism developed here together with the work of Thimm to give a very explicit proof of complete integrability of the geodesic flow on symmetric spaces such as the real and complex Grassmannians.

Interestingly, as a special case of this approach we were able to derive a new symmetric form of the classical rigid body equations.

The equations are of the form:

$$\begin{aligned} \dot{Q} &= \Omega Q \\ \dot{P} &= \Omega P \end{aligned} \tag{1.8}$$

where Q is the configuration matrix of the body and $\Omega = J^{-1}(QP^T - PQ^T)$ is the angular velocity.

We were able to relate these equations (see Bloch, Crouch, Marsden and Ratiu [1998]) to the discrete rigid body equations of Moser and Veselov and hence begin work on a new numerical algorithms for rigid bodies.

In Bloch and Crouch [1998b] we analyzed the structure of interconnected systems including electrical circuits.

With Alan Markworth (Markworth and Bloch [1996, 1996a]) I examined the control of systems associated with anti-corrosion processes. The free system contains, in general, rather complex behavior such as periodic orbits or chaotic attractors. We were able to stabilize this system using classical control methods as well as more modern methods based on my work with Marsden on stabilizing nonlinear systems with homoclinic or heteroclinic orbits.

With N. McClamroch and a student Rui [1997] we considered stability of a satellite with robot arm and momentum wheel. Numerics were done using the ADAMS software from Mechanical Dynamics.

Executive Summary

Personnel Supported, Theses Arising, Publications

Personnel Supported:

Students K. Lum, D. Zenkov and postdoc M.Gekhtman.

Theses Supervised:

K. Lum: Control of the Rigid Body and Dynamics with Symmetry. Ph.D Dissertation, Aerospace Engineering, The University of Michigan.

Contains: work on global stabilization of spinning top with mass imbalance, adaptive virtual autobalancing of rotor with mass balance, Serret-Andoyer transformation of variables for the controlled rigid body, reduction of full system on $T^*SO(3)$, proof of stabilization of the rigid body using a rotor, geometric approach to output feedback regulation.

D. Zenkov: Integrablity and Stability of Nonholonomic Systems, Ph.D Dissertation, Department of Mathematics, The Ohio State University.

Contains: Analysis of the Routh Problem of a rigid body rolling on a surface of revolution, proof that this problem is completely integrable, nonlinear conditions for stability of motion, energy momentum approach extended to nonholonomic systems including those with internal dissipation, i.e. demonstration we can find a Lyapunov function of the form $V = E + \Phi$ where E is the energy and Φ is another conserved quantity, Φ shown not to be the momentum in general but a quantity which arises from the analysis of the dynamic momentum equation, applications to the coupled wheeled vehicles and the rattleback top.

More details in the publications below and in the technical write-up above.

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